Interplay between spin-relaxation and Andreev reflection in ferromagnetic wires with superconducting contacts

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(today)

We analyze the change in the resistance of a junction between a diffusive ferromagnetic (F) wire and normal metal electrode, due to the onset of superconductivity (S) in the latter and a double Andreev scattering process leading to a complete internal reflection of a large fraction of the spin-polarized electrons back into the ferromagnet. The superconducting transition results in an additional contact resistance arising from the necessity to match spin-polarized current in F-wire to spin-less current in S-reservoir, which is comparable to the resistance of a piece of a F-wire with the length equal to the spin-relaxation length.

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Recent advances in microprocessing technologies of metals have allowed one to obtain high quality nanostrucutres formed from combinations of superconductoring (S) and ferromagnetic (F) materials[1−5]. In such structures one may expect to see manifestations of Andreev reflection[6] at the FS interface whereby electrons with excitation energies $\varepsilon \lesssim \Delta$ from a normal (N) metal convert into a hole with the opposite spin, and the normal current in the N-part of a circuit transforms into supercurrent inside the superconductor. Both theoretical and experimental studies of subgap transport of NS structures have revealed that Andreev reflection may substantially change the circuit resistance - by the values comparable to the resistance of a mesoscopic length segment of the normal metal wire[7−16]. The difference between the NN and NS resistances, $R_{\rm N}$ and $R_{\rm S}$, is determined by the extent of the proximity effect over the non-superconducting material. In the presence of a large exchange field $\varepsilon_{ex} \gg \Delta$, electron-hole correlations are suppressed in ferromagnets on a microscopic length scale, so that Andreev processes at the FS interface do not generate subgap conductance effects related to condensate penetration into the non-superconducting part of the circuit.

However, there exists another mechanism of large conductance variations below the critical temperature T_c in FS structures, which has a classical nature and arises from the necessity to match the spin-polarized electron current at one end of the ferromagnetic lead to the spinless current inside the superconductor, whereas above T_c the boundary conditions allow the spin-polarized current to flow through the entire circuit. The degree of spin polarization, $\varsigma = (j_\uparrow - j_\downarrow)/(j_\uparrow + j_\downarrow)$ carried by the electric current in a free standing single magnetic domain F-wire,

$$\varsigma = \left(D_+\nu_+ - D_-\nu_-\right)/\left(D_+\nu_+ + D_-\nu_-\right), \qquad (1)$$

may be substantial if Fermi-surfaces of spin-up and spin-down electrons are very different (e.g., $D_+\nu_+ \ll D_-\nu_-$). Here, $D_\alpha = \frac{1}{3}v_\alpha l_\alpha$ and ν_α are the diffusion coefficients and densities of states in the ferromagnet, $\alpha = \pm$ describe spin 'up' and 'down', v_α are the Fermi velocities, l_α - the mean free paths. Note that we are interested in a situation where the resistivity of ferromagnet dominates the circuit resistance, and the resistance of the S-part of the system can be neglected even when it is in the normal state. In the absence of spin-relaxation, the only solution to this problem consists of assuming a slight non-equilibrium (current-induced) spin-repolarization of the ferromagnetic wire, so that the diffusion of locally accumulated non-equilibrium spin-density would compensate the spin carried by the electric current. That is, everywhere across the F-wire, the local values of chemical potentials of spin-up and -down electrons split, which limits the conduction by that of the worst conducting spin-state. By taking into account both the electron and Andreev-reflected hole currents, this logical exercise may be upgraded to yield an estimate of the resistance variation of a spin-conserving F-wire, $(R_{\rm S} - R_{\rm N})/R_{\rm N} = (D_+\nu_+ - D_-\nu_-)^2/4(D_+\nu_+D_-\nu_-) \equiv \varsigma^2/(1-\varsigma^2)$, which can be also deduced from the result obtained by de Jong and Beenakker[17] using the Landauer-Buttiker approach extended to the hybrid NS structures[12,14].

In this paper, we study the evolution of the resistance of a ferromagnetic wire under the superconducting transition in the bulk electrode with emphasis on the case of a diffusive F-wire whose length is comparable to, or much longer than the length L_s of spin-relaxation processes. The resistance of a macroscopically long wire (with cross-section $L_\perp \times d$ and resistance per square $R_\square^{-1} = e^2(D_-\nu_- + D_+\nu_+)d$), can be split into a 'bulk' part and a contact resistance $r_{\rm c}$ formed within mesoscopic region near the FN or FS junction, such that $R = (L/L_\perp)\,R_\square + r_{\rm c}$. As we find below, the contact resistance $r_{\rm c}$ (which, in the normal state, is determined by the relation between Fermi surfaces of carriers in F and N) acquires below T_c an additional contribution equal to the resistance of a segment of the F-wire with the length L_s. Below, we present the semiclassical analysis of this effect, which includes calculation of the classical resistance variations near T_c and down to the zero-temperature limit, and an estimate of the weak localization correction to it.

The resistance of a disordered F-wire can be found

by solving diffusion equations for the isotropic part of the electron distribution function, $n_\alpha(z,\varepsilon) = \int d\Omega_{\mathbf{p}} n_\alpha(z,\mathbf{p})$. Due to the electron-hole symmetry, and in order to simplify the calculation of the FS case, we shall compute the symmetrized function $N_\alpha(\varepsilon,z) = \frac{1}{2}\left[n_\alpha(z,\varepsilon)+n_\alpha(z,-\varepsilon)\right]$, where ε is determined with respect to the chemical potential in the S(N) electrode. In terms of $N_\alpha(\varepsilon,z)$, the current density is given by $j_\alpha = e^2\nu_\alpha D_\alpha \int_0^\infty d\varepsilon \partial_z N_\alpha(\varepsilon,z)$, and $N_\alpha(\varepsilon,z)$ obey the diffusion equation

$$D_\alpha \partial_z^2 N_\alpha(z,\varepsilon) = w_{\uparrow\downarrow}\nu_{-\alpha}\left[N_\alpha(z,\varepsilon) - N_{-\alpha}(z,\varepsilon)\right], \quad (2)$$

which is more convenient to use in the equivalent form

$$\partial_z^2 \sum_{\alpha=\pm} D_\alpha \nu_\alpha N_\alpha = 0,\ \left[\partial_z^2 - L_s^{-2}\right](N_+ - N_-) = 0. \quad (3)$$

The term on the right hand side of Eq. (2) accounts for spin-relaxation which may result from both spin-orbit scattering at defects and the surface, and from the random weak precession of electron spins when passing through adjacent non-collinearly magnetized ferromagnetic domains. It can be used to define the effective spin-relaxation length, L_s as $L_s^{-2} = w_{\uparrow\downarrow}\left[\nu_\uparrow/D_\downarrow + \nu_\downarrow/D_\uparrow\right]$. This pair of equations, which ignore any energy relaxation, should be complemented by four boundary conditions, two on each side of the ferromagnetic wire.

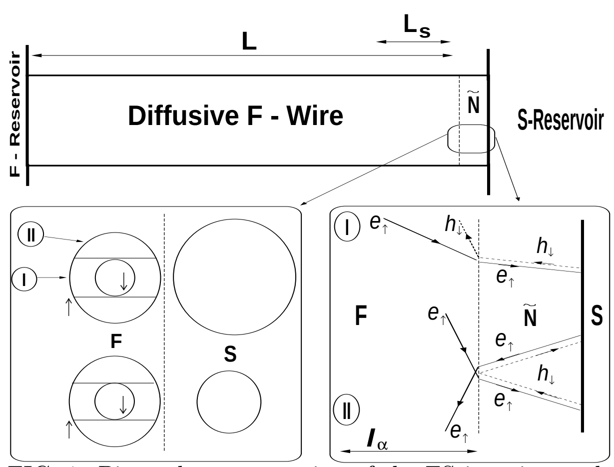

FIG. 1. Pictural representation of the FS junction, a double Andreev reflection processes in it, and of possible relations between Fermi surfaces of spin-up and -down electrons in the F-wire (left) and in the N/S metal (right)

To obtain boundary conditions for Eqs. (2) and their solutions in a specific geometry, we employ the model shown in Fig. 1, where the FS junction is replaced by a sandwich of three layers: (i) a ferromagnetic (F) wire of the length L connected to the bulk F reservoir, (ii) a normal metal layer ($\widetilde{N}$) which never undergoes a superconducting transition by itself and has a negligible resistance, and (iii) a bulk electrode S(N) which undergoes the superconducting transition. The insertion of a normal metal layer $\widetilde{N}$ between the F and S(N) parts allows us to formulate the boundary conditions at the FS interface using known boundary conditions at the $\widetilde{N}$S interface[14,18]. For the sake of simplicity, we consider $\widetilde{N}$ as ballistic and the F$\widetilde{N}$ junction - as semiclassically transparent, so that electrons either pass from one side to the other, or are fully reflected, depending on whether this process is allowed by the energy-momentum conservation near the Fermi surface. The latter approximation avoids resonances through the 'surface states'[19] due to multiple passage through the normal layer inserted between S and F. As illustrated in Fig. 1, we approximate the spectrum of electrons by parabolic bands - two for spin-down and spin-up electrons in F, and one in the N-part, which we take into account by introducing the parameters $\delta_{\alpha\mathrm{N}}^2 = p_{\mathrm{FN}}^2/p_{\mathrm{F}\alpha}^2$ and $\delta^2 = \left(p_{\mathrm{F}-}/p_{\mathrm{F}+}\right)^2 < 1$. The $\widetilde{N}$N interface is assumed to be ideal, and the Fermi surfaces in $\widetilde{N}$ and N layers to be the same, so that $\widetilde{N}$S Andreev reflection has unit probability. In such a model, the momentum of an electron in the plane of the junction is conserved.

The boundary conditions on the left end are given by the equilibrium distribution of electrons in the F-electrode, $N_\alpha(-L,\varepsilon) = \frac{1}{2}\left[n_T(\varepsilon - eV) + n_T(-\varepsilon - eV)\right]$. The boundary condition on the other end depends on the state of the electrode, and in the superconducting state takes into account Andreev reflection at the NS interface[6]. Since in our model of an ideal F$\widetilde{N}$ interface, the parallel component of the electron momentum is conserved, the effective reflection/transmission of electrons in parts I and II of the ferromagnet Fermi surface sketched in Fig. 1 are different. Although non-equilibrium quasi-particles from F pass inside $\widetilde{N}$ and generate holes by being Andreev reflected at the $\widetilde{N}$S interface, only those holes which are created by quasi-electrons from part I of the Fermi surface in F may escape into the F-wire. The spin-down holes which were generated by spin-up electrons from part II of the Fermi surface cannot find states in F, so that they are fully internally reflected into $\widetilde{N}$. Then, they undergo a second Andreev reflection, convert into the spin-up electrons, and return back into the ferromagnetic wire. This results in *complete internal reflection* of spin-up electrons from part II of the Fermi surface inside the F-wire, which nullifies the spin current through its FS edge.

The boundary condition near the F$\widetilde{N}$ junction can be found by matching the isoenergetic electron fluxes determined in the diffusive region found in the ballistic F-region using the reflection/transmission relation between the distributions of incident and Andreev or normal reflected electrons. The algebraic procedure used in this derivation is sketched in footnote[20]. For quasi-particles with energies $0 < \varepsilon < \Delta$ this can be written in the form

$$D_+\nu_+\partial_z N_+ - D_-\nu_-\partial_z N_- = 0, \quad (4)$$

$$N_+ + N_- + \frac{2}{3}\varkappa\delta^2 l_- \partial_z N_- = 2N_T(\varepsilon), \quad (5)$$

where $\varkappa = (1 - \delta^2)^{3/2}/\delta^2$, $\delta^2 = p_{\mathrm{F}-}^2/p_{\mathrm{F}+}^2 < 1$, and $N_T(\varepsilon) = \frac{1}{2}\left[n_T(\varepsilon) + n_T(-\varepsilon)\right] = \frac{1}{2}$. A similar result[21] can be derived by considering the inserted $\widetilde{N}$-layer as diffusive, if we employ the known boundary conditions from Refs.[13,14,22]. At $\varepsilon > \Delta$ the boundary conditions are the same as in the normal state, $N_\pm = \frac{1}{2}$.

By solving them at low temperatures, $T \ll T_c$, we arrive at the contact resistance of the FS boundary

$$r_{\mathrm{c}}^{\mathrm{S}} = R_\Box \frac{L_s}{L_\perp} \frac{\varsigma^2}{1 - \varsigma^2} + \frac{R_\Box l_+}{3L_\perp} \frac{\varkappa}{1 + \varsigma}. \tag{6}$$

In the normal state of the right hand reservoir, the boundary conditions at the end of F-wire depend on the relation between the Fermi momenta of electrons in the ferromagnet and normal metal,

$$N_\alpha(z, \varepsilon) + \frac{4\varkappa_{\alpha\mathrm{N}} D_\alpha}{v_\alpha} \partial_z N_\alpha(z, \varepsilon)\Big|_{z=0} = N_T(\varepsilon),$$

where $\varkappa_{\alpha\mathrm{N}} = (1 - \delta_{\alpha\mathrm{N}}^2)^{3/2}/\delta_{\alpha\mathrm{N}}^2$, $\delta_{\alpha\mathrm{N}} < 1$, and $\varkappa_{\alpha\mathrm{N}} = 0$, $\delta_{\alpha\mathrm{N}} \geq 1$, $\delta_{\alpha\mathrm{N}}^2 = p_{\mathrm{FN}}^2/p_{\mathrm{F}\alpha}^2$. These result in the contact resistance term

$$r_{\mathrm{c}}^{\mathrm{N}} = R_\Box \frac{l_+ (1 + \varsigma)}{L_\perp} \left\{ (1 - \varsigma)\, l_+/L_s + \frac{3}{2} \varkappa_{+\mathrm{N}}^{-1} \right\}^{-1}, \tag{7}$$

which has sense only when it is larger than the resistance of the short piece of the F-wire with the length of the order of l_+ and otherwise should be neglected.

After comparing the latter result to $r_{\mathrm{c}}^{\mathrm{S}}$, we find that the resistance of a long ferromagnetic wire attached to the S-electrode exceeds the resistance of the same wire connected to the normal reservoir by the resistance of a F-segment of length of order of L_s. One can extend the result of Eq. (6) to finite temperatures, which yields the resistance variation below the superconducting transition

$$R_{\mathrm{S}}(T) - R_{\mathrm{N}} \approx \frac{\varsigma^2}{1 - \varsigma^2} \frac{L_s}{L_\perp} R_\Box \tanh\left(\frac{\Delta(T)}{2T}\right). \tag{8}$$

The increase of the resistance in Eq. (8) originates from *the matching of a spin-polarized current in the highly resistive ferromagnetic wire to a spinless current inside the superconductor*, and represents the main result of this paper. This robust classical effect is peculiar to monodomain wires, with the domain size $L_{\mathrm{D}} > L_s$. In a multidomain wire, with a finely coarse-grained collinear magnetic structure, the transport properties of spin-up and down electrons do not differ, $\varsigma \to 0$, the spin-current in the bulk of the F-wire is equal to zero, and, therefore, the classical contact resistance effect described by Eq. (8) is absent.

When speaking about the opposite limit of a short wire with $L \ll L_s$, it is more appropriate to discuss the conductance variation of the entire wire, $G_{\mathrm{N}} - G_{\mathrm{S}}$ rather than contact resistances, which can be represented as

$$\frac{G_{\mathrm{S}}(T) - G_{\mathrm{N}}}{G_{\mathrm{N}}} \approx \varsigma^2 \tanh\left(\frac{\Delta(T)}{2T}\right). \tag{9}$$

In addition to the effect described by Eq. (8) which originates from a mesoscopic region of the F-wire, Eqs. (6) and (7) also contain a small contribution to the variation $r_{\mathrm{c}}^{\mathrm{S}} - r_{\mathrm{c}}^{\mathrm{N}}$ from 2e charge transfer at the FS interface, due to the Andreev process. The latter contribution to the classical resistance should be taken into account only if its value greatly exceeds $R_\Box l_+/L_\perp$, and, for $\varsigma = 0$, it has the form $\lim_{\varsigma\to 0}(R_{\mathrm{S}} - R_{\mathrm{N}}) \approx [\varkappa - 2\varkappa_{+\mathrm{N}}]\, R_\Box l_+/3L_\perp$, similar to de Jong and Beenakker's result for the FS ballistic point contact[17]. In contrast with the added contact resistance in Eq. (8), this may have an arbitrary sign, depending on the ratios δ^2 and $\delta_{+\mathrm{N}}^2$ between the areas of Fermi surfaces of electrons in the normal metal and ferromagnet (for the system illustrated by Fig. 1, $\varkappa > \varkappa_{+\mathrm{N}}$).

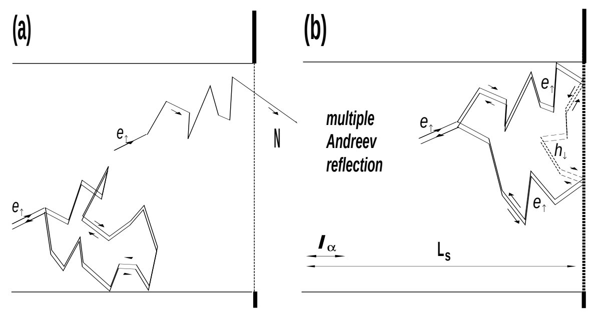

FIG. 2. (a) Cooperon decay at the FN boundary due to the electron escape into the normal reservoir. (b) Change of the boundary conditions for the Cooperon due to a multiple Andreev reflection.

As wellas the above classical resistance effect, one may discuss the variation of the weak localization correction to the contact resistance, in particular, in wires with a finely coarse-grained collinear magnetic structure, where Eq. (8) gives zero effect. Since the weak localization correction can be easily destroyed by a magnetic field, we have to assume that the wire cross-section is small and the intrinsic magnetic field in it is not large enough to suppress the enhanced back scattering effect. To calculate the weak localization correction, $\Delta R_{\mathrm{S}} - \Delta R_{\mathrm{N}}$ to the variation of the contact resistance in the latter situation, one should take into account the following features of the problem: (a) The weak localization correction to the resistance, is dominated by the triplet channel of the Cooperon, where the Cooperon spin projection onto the overall magnetization direction is $m = \pm 1$, and is restricted within the length scale L_s (we assume that $L_s > L_\perp$). (b) The weak localization effect in the conductance is affected by the change of the boundary conditions for the Cooperon due to the multiple Andreev process, as illustrated in Fig. 2. In the normal state, the electrons escape to the electrode, whereas in the S-state they undergo several Andreev reflections, so that they may return to the same point carrying the initial spin and contribute to interference. As a result, the resistance of the entire circuit changes (between T_c and a low temperature) by an amount corresponding to the change

of the conductance of the last L_s-segment of the F-wire:

$$\Delta R_{\rm S} - \Delta R_{\rm N} \sim \frac{e^2}{h}\left(R_\Box \frac{L_s}{L_\perp}\right)^2 .$$

The details of analogous estimations for a single-domain F-wire will be reported elsewhere.

In conclusion, we have calculated the resistance variation of an FS structure below the critical temperature and shown that the sign and magnitude of this effect crucially depends on the domain structure of the ferromagnetic part of the circuit. In a single domain F-wire with length larger than spin-relaxation length L_s, this variation has classical origin and is formed within the last L_s-segment of the wire (where the spin-polarized current brought from the F-part relaxes into spin-less current in a superconductor), and $R_{\rm S}(T) - R_{\rm N}$ increases from zero at T_c to a positive value at $T = 0$. In a coarse-grained multi-domain wire, $R_{\rm S}(T) - R_{\rm N}$ is determined by the interplay between the mismatch of Fermi-surfaces and the weak localization effect, and may have an arbitrary sign.

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[1] V.T.Petrashov *et al*, JETP Lett. **59**, 551 (1994); V.T. Petrashov *et al*, preprint

[2] H.D.Lawrence and N.Giordano, J. Phys. Cond. Matt. **8**, L563 (1996)

[3] M.Giroud, H.Courtois, K.Hasselbach, D.Mailly and B.Pannetier, Phys. Rev. B **58,** 11872 (1998)

[4] V.A.Vasko et al, Phys. Rev. Lett. **78**, 1134 (1997)

[5] S.K.Upadhyay, A.Palanisami, R.N.Louie, R.A.Buhrman, Phys. Rev. Lett. **81**, 3247 (1998)

[6] A.F. Andreev, Zh. Exp.Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)]

[7] V.T. Petrashov *et al*, Phys. Rev. Lett. **70**, 347 (1993); **74**, 5268 (1995)

[8] P.G.N.Vegvar *et al*, *ibid.* **73**, 1416 (1994)

[9] H.Pothier, S.Gueron, D.Esteve, M.H.Devoret, Phys. Rev. Lett. **73**, 2488 (1994)

[10] H.Dimoulas *et al*, Phys. Rev. Lett. **74**, 602 (1995); S.G. den Hartog *et al*, *ibid.* **76**, 4592 (1996)

[11] H.Courtois. P.Grandit, D.Mailly, B.Pannetier, Phys. Rev. Lett. **76**, 130 (1996); D.Charlet *et al*, *ibid.* **77**, 4950 (1996)

[12] C.W.J.Beenakker, Rev. Mod. Phys. **69**, 731 (1997)

[13] S.N.Artemenko, A.F.Volkov, Sov.Phys. Usp. **22**, 295 (1979)

[14] C.Lambert, R.Raimondi, J. Phys. Cond. Matt. **10** (1998)

[15] F.W.Hekking and Yu.Nazarov, Phys. Rev. Lett. **71**, 1725 (1993); Yu.Nazarov and T.H.Stoof, *ibid.* **76**, 823 (1996)

[16] A.F.Volkov, A.V.Zaitsev, T.M.Klapwijk, Physica C**210**, 21 (1993); A.V.Zaitsev, JETP **51**, 41 (1990)

[17] M.J.M. de Jong, C.W.J.Beenakker, *ibid.* **74**, 1657 (1995)

[18] A.I.Larkin and Yu.N.Ovchinnikov, p.493 in *Non-Equilibrium Superconductivity*, ed. by D.N.Langenberg and A.I.Larkin, Elsevier, Amsterdam 1986

[19] S. Kuplevakhski and Igor I. Falko, JETP Lett. **52**, 957 (1990); Phys. Met. Metallogr. (USSR) **71**, 68 (1991)

[20] To match the isoenergetic electron fluxes determined in the diffusive region, we use relations between the distributions of reflected electrons, $\widetilde{n}_\alpha(z,\mathbf{p}_\parallel,p_z<0)$ defined only for $p_z<0$ and the distribution of electrons approaching the F$\widetilde{N}$ interface from the diffusive region, $n_\alpha(z,\mathbf{p}_\parallel,p_z) = n_\alpha(z,\varepsilon) - l_\alpha \partial_z n_\alpha(z,\varepsilon)$:

$$\int_{p_z<0} d\mathbf{p} v_\alpha^z \left[n_\alpha(\mathbf{p},0) - \widetilde{n}_\alpha(\mathbf{p},0)\right]\delta(\varepsilon - E_\alpha(\mathbf{p})) = 0.$$

This is the same both in the case of F$\widetilde{N}$N and F$\widetilde{N}$S contacts; in the second case, the boundary condition relates electron distributions at the energies $\pm\varepsilon$:

$$\begin{aligned}&\left[\widetilde{n}_\alpha(\mathbf{p}_\parallel,|p_\alpha^z|,\pm\varepsilon) - n_T(\pm\varepsilon)\right]\\ &= \frac{v_{-\alpha}^z}{v_\alpha^z}\frac{dp_{-\alpha}^z}{dp_\alpha^z}\left[n_T(\mp\varepsilon) - n_{-\alpha}(\mathbf{p}_\parallel,p_\alpha^z>0,\mp\varepsilon)\right],\end{aligned}$$

for the part I of the Fermi surface, and, in the part II - as

$$\widetilde{n}_+(\mathbf{p}_\parallel,p_+^z,\varepsilon) = \widetilde{n}_+(\mathbf{p}_\parallel,p_+^z>0,\varepsilon).$$

Note that in the above equations $|p_+^z|$ and $|p_-^z|$ are related through $E_+(\mathbf{p}_\parallel,p_+^z) = E_-(\mathbf{p}_\parallel,p_-^z)$.

[21] In particular, Eq. (4) is analogous to the Larkin-Ovchinnikov boundary condition at the NS interface, $\partial_z\left[n_\pm(\varepsilon) - n_\mp(-\varepsilon)\right] = 0$.

[22] V.I. Falko, A.F. Volkov and C.Lambert, unpublished